Clique-covering co-bridge-free prismatic graphs $\iff$ Coloring bridge-free antiprismatic graphs

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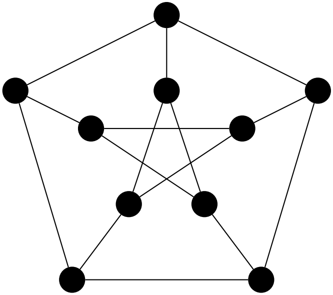
IRIF - Université Paris Cité

Joint work with Eileen Robinson
Université libre de Bruxelles

GRIF - 2026

Coloring problem



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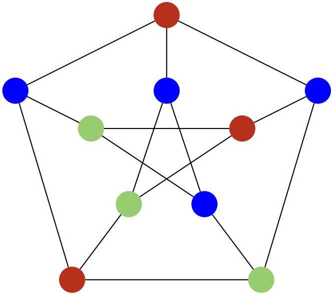

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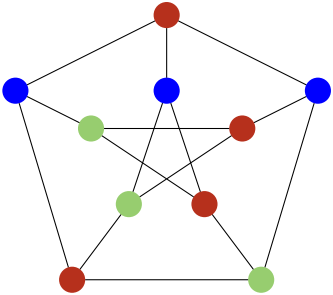

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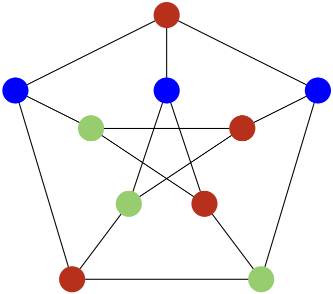

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NP-Complete in the general case

Hereditary classes of graph

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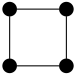
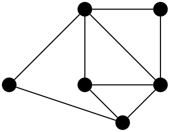
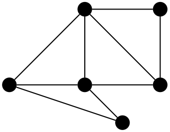


C_4



G_1



G_2

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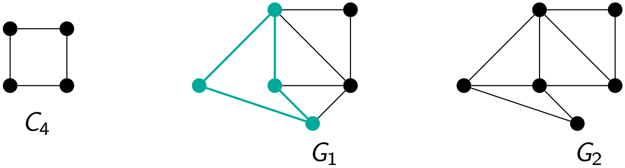

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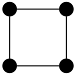

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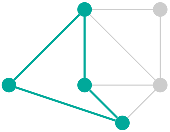

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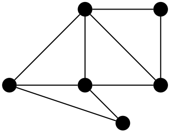

G_2

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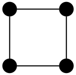

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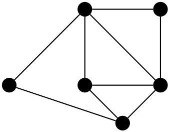

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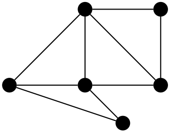

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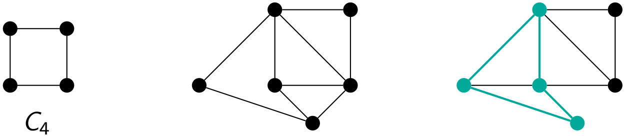

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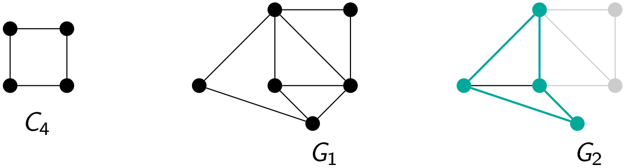

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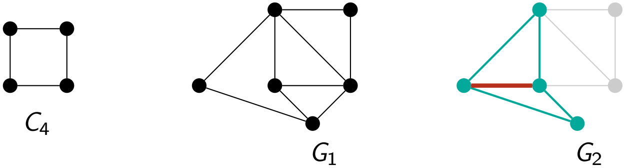

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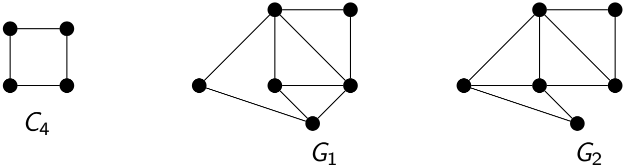

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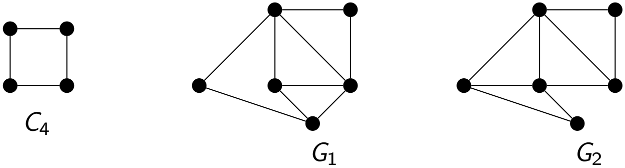

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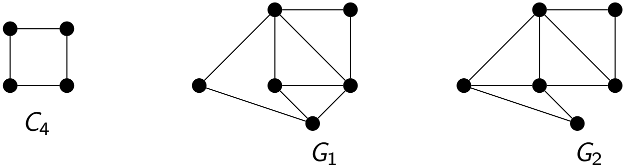

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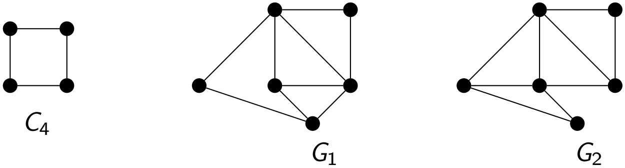

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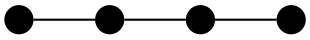
Complexity of the coloring problem

Theorem: Král, Kratochvíl, Tuza and Woeginger [2001]

The Coloring problem is polynomial-time solvable for H-free graphs if H is an induced subgraph of P_4 or of $P_3 + K_1$ and it is NP-complete for any other H.



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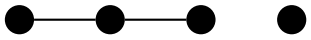

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Lozin and Malyshev: when excluding graphs of order 4.

Open Cases:

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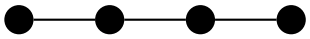

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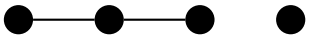

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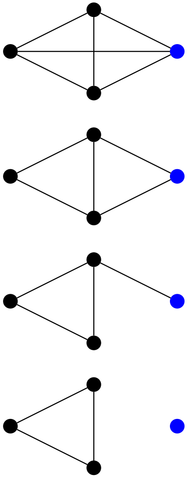
Property: Lozin and Malyshev

Complexity of Vertex Coloring $\mathcal{F}_2$
$\Leftrightarrow$ Complexity of Vertex Coloring $\mathcal{F}_3$.

Prismatic graphs

Definition: Prismatic graphs

A graph G is prismatic if $\forall$ triangle T in G, every vertex v of G that is not in T has exactly one neighbour in T.



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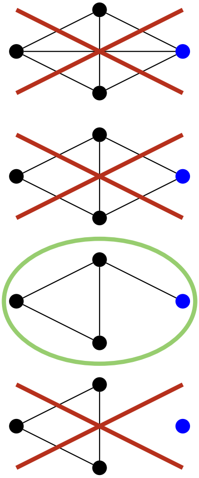

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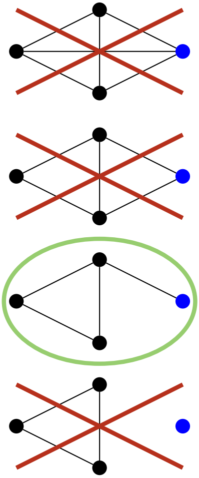
$\Leftrightarrow$ Complement graphs of $\mathcal{F}_3$



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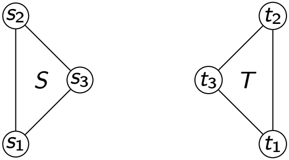

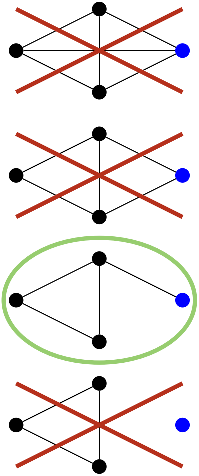

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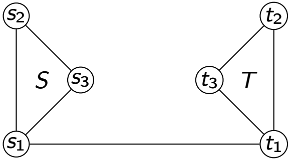

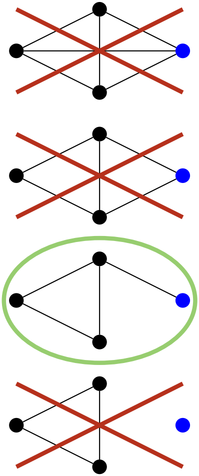

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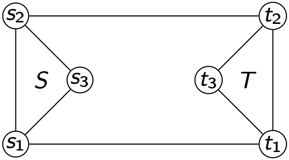

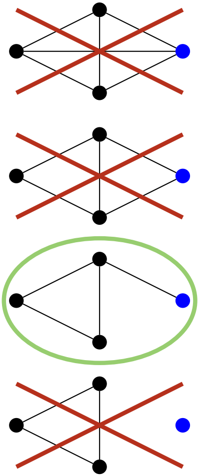

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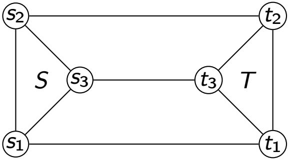

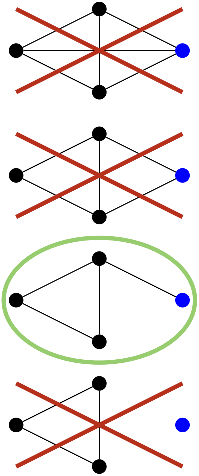

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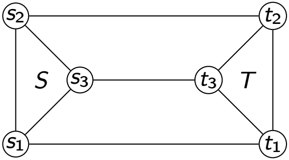
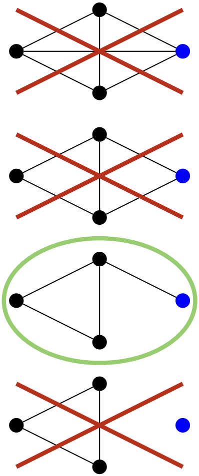


Prism



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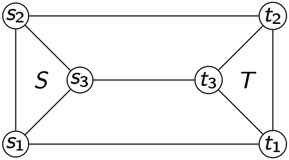
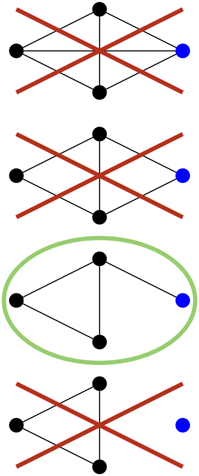
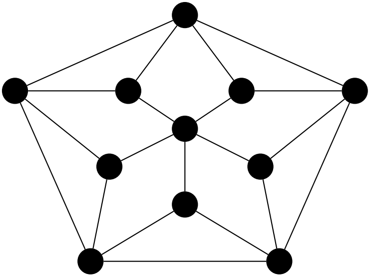




Coloring Problem in $\mathcal{F}_3$
$\Leftrightarrow$ Clique Covering Problem in Prismatic graphs.

Clique Covering Problem



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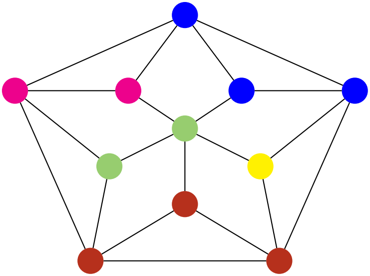

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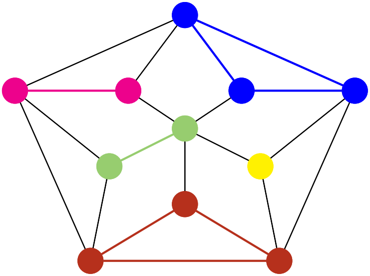

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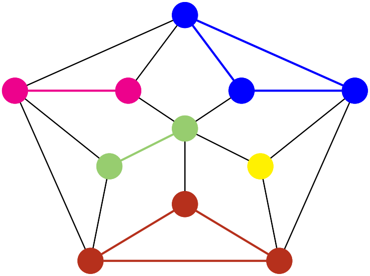

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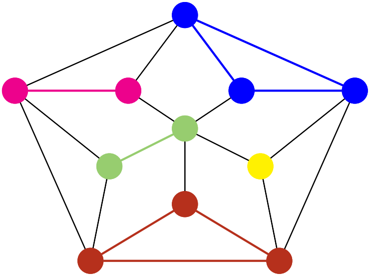

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Coloring graphs in $\mathcal{F}_3 \Leftrightarrow$ Clique covering prismatic graphs.

Structure of Prismatic graphs

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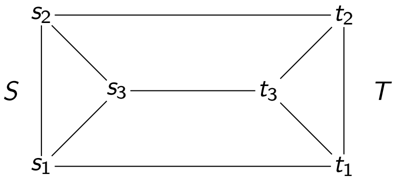

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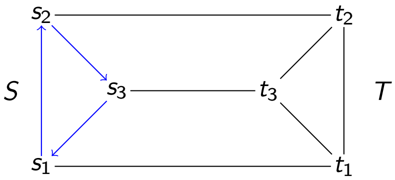

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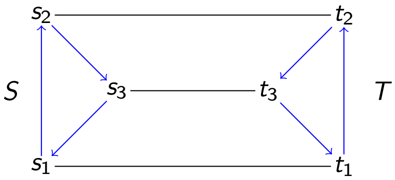

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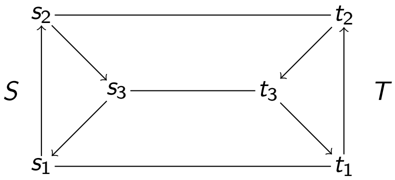

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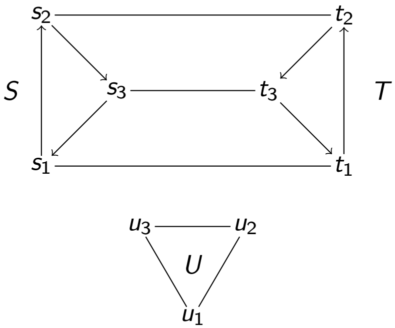

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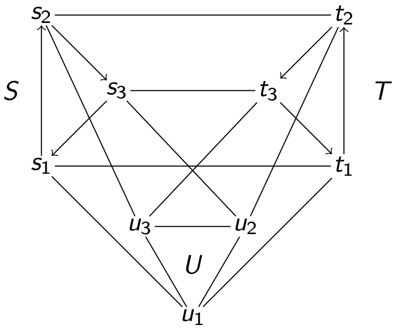

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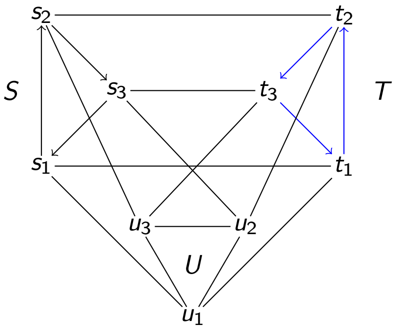

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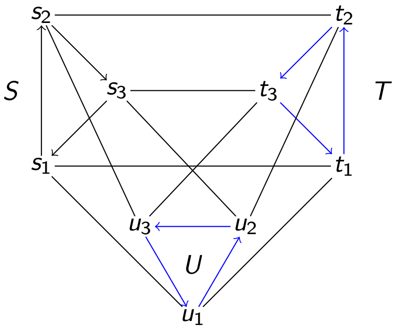

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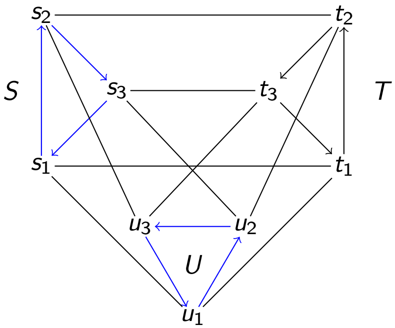

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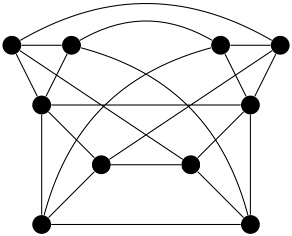

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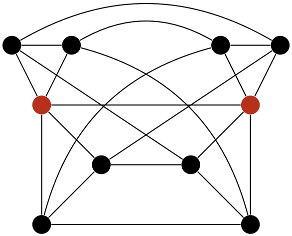

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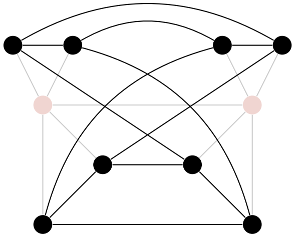

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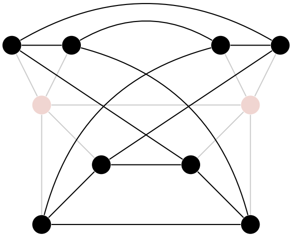

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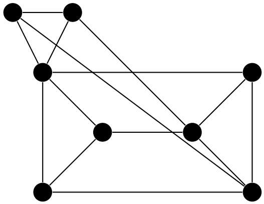

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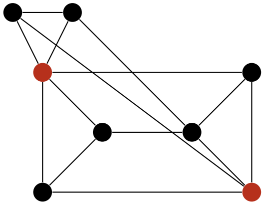

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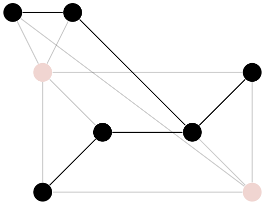

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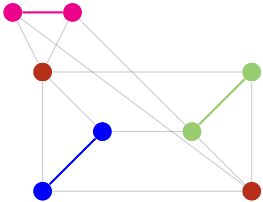

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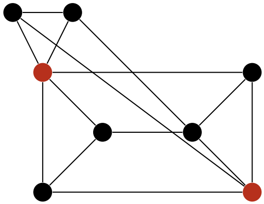

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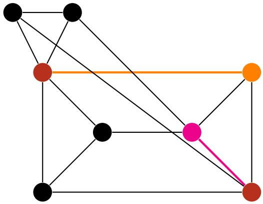

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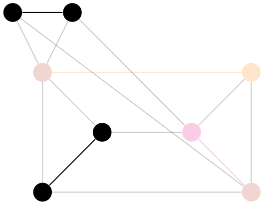

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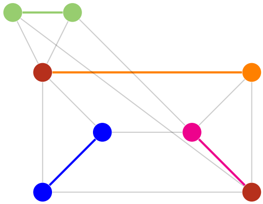

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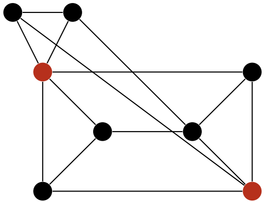

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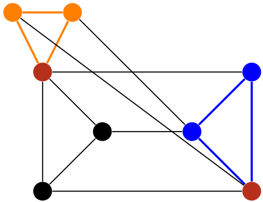

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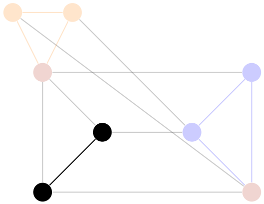

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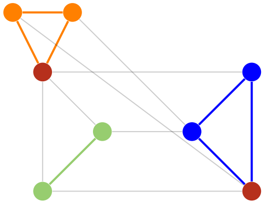

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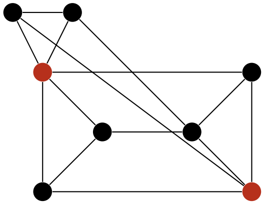

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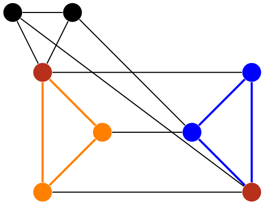

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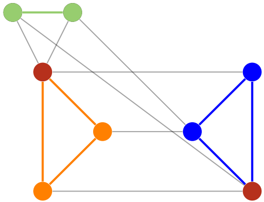

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Theorem [Preissmann, Robin and Trotignon (2021)

Every non-orientable prismatic graph either:

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- or has at most 27 vertices (Schläfli-prismatic).

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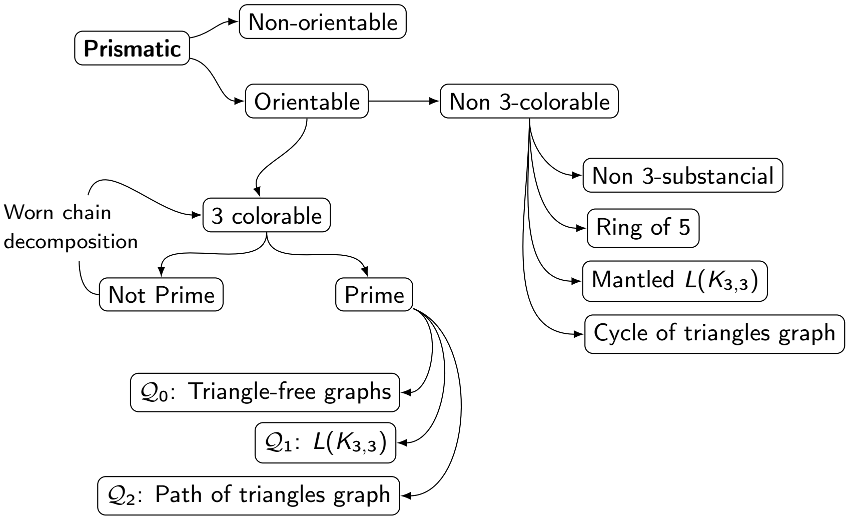
Why doesn't it work in the orientable case?

$\Rightarrow$ To have a lot of disjoint triangles, organize the triangle. $\Rightarrow$ be orientable.

Strucutre of prismatic graph



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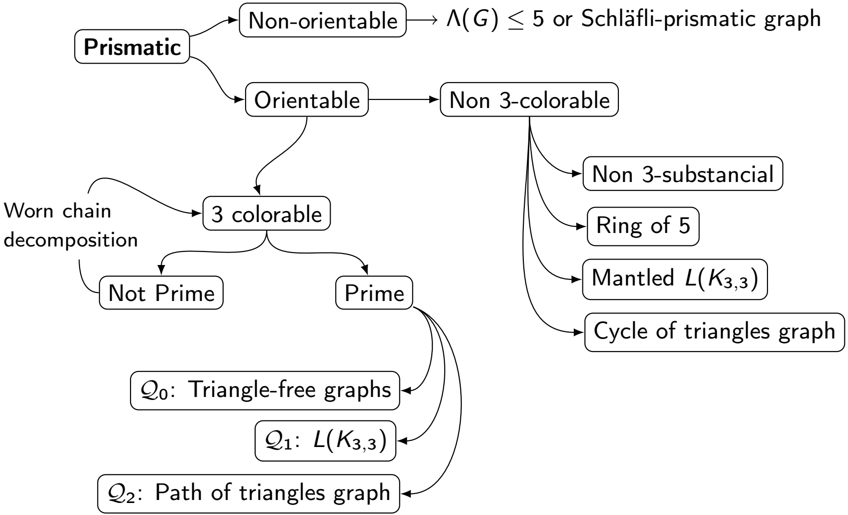

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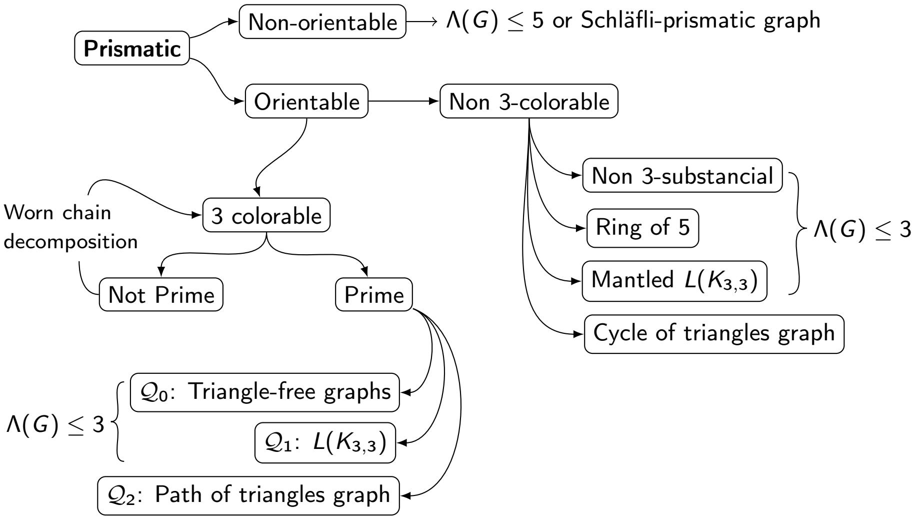

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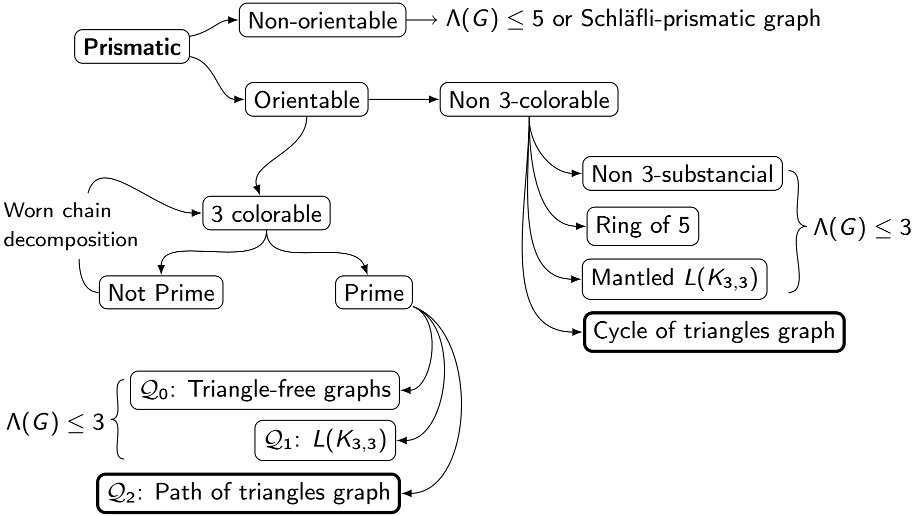
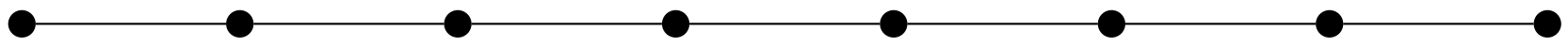


Path of triangles graphs



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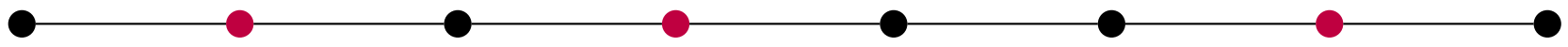

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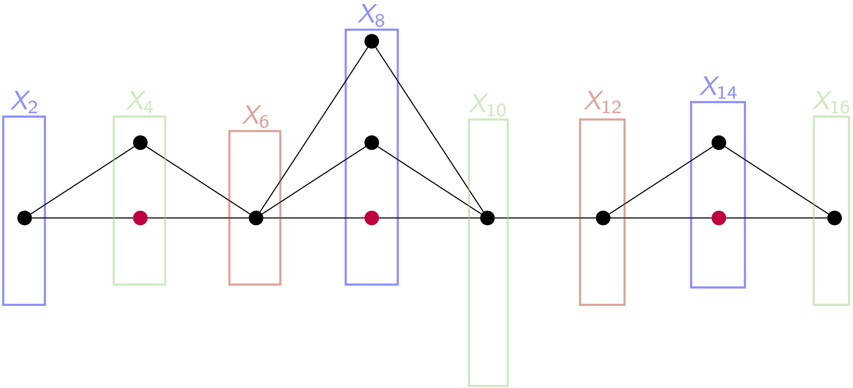

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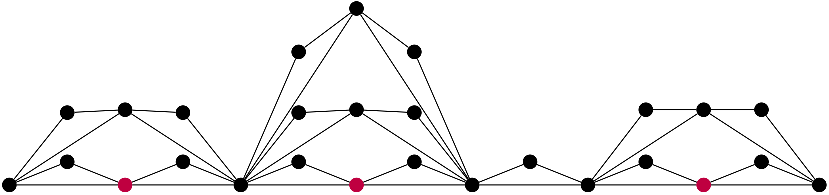

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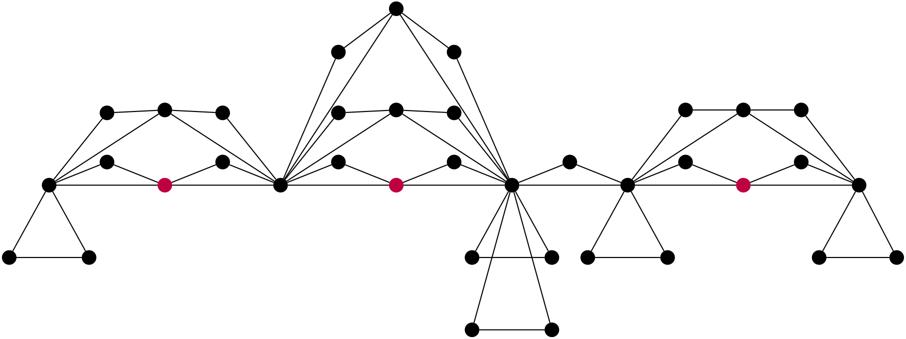

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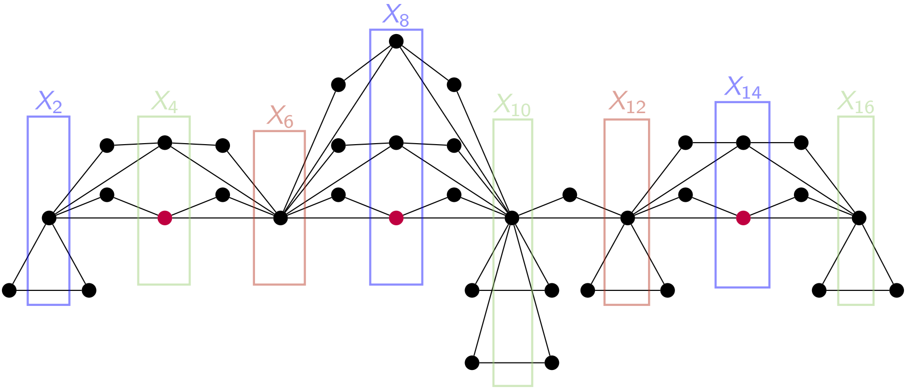

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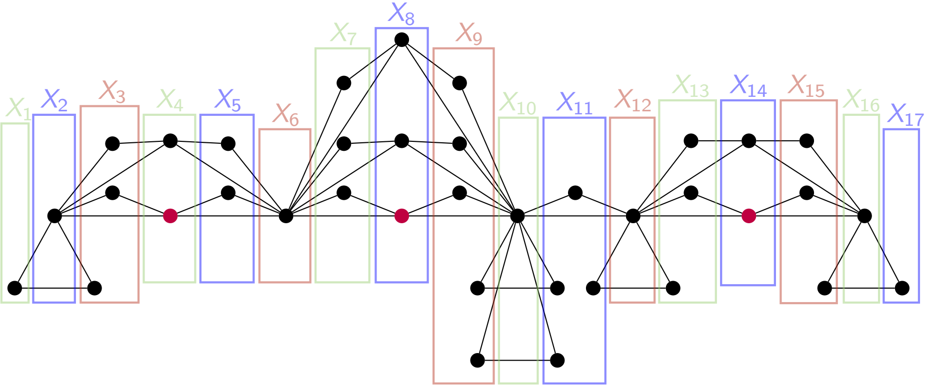

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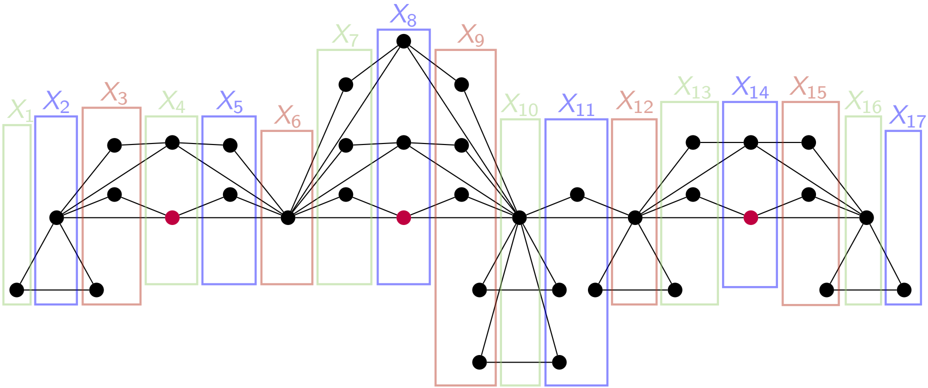

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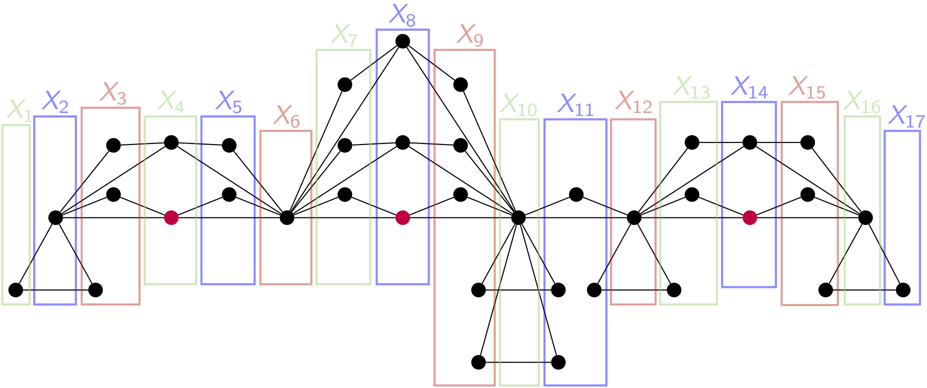
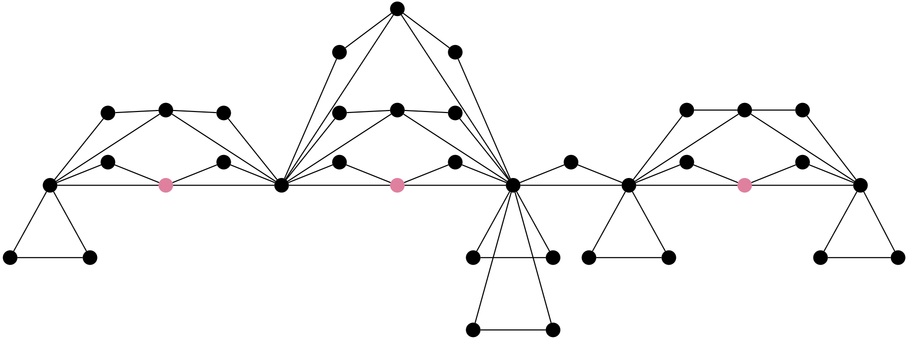


Cycle of triangle graphs = start with cycle of length $3k + 2$.

Hitting set for Path of triangles graphs

Those are the only triangles in a path of triangles graphs.



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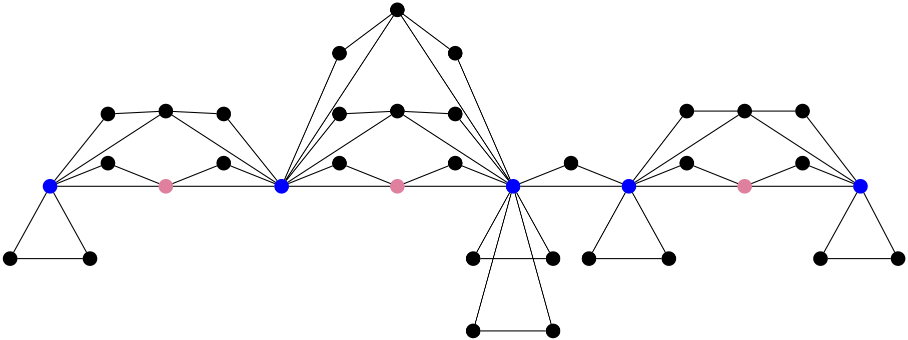

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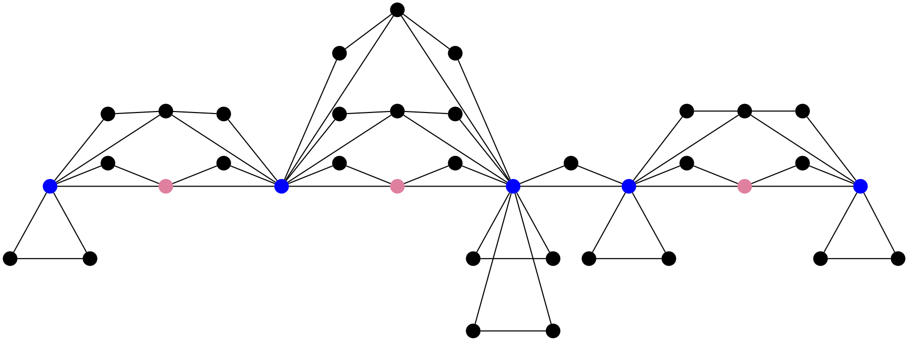

| Hitting set | $\leq$ size of the path.

Hitting set for co-bridge-free prismatic graphs

Theorem [Robin and Robinson (2025)]

Every co-bridge-free prismatic graph either:

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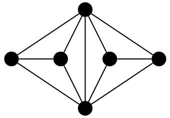
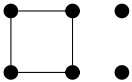


Bridge



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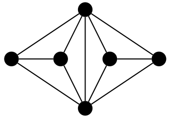
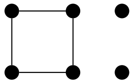


Bridge

Ideas for the proof

- Paths and Cycles of triangles graphs can not be too long.
- It is not possible to add too many triangles with the worn chain decomposition.



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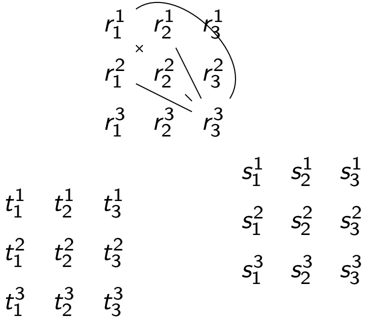

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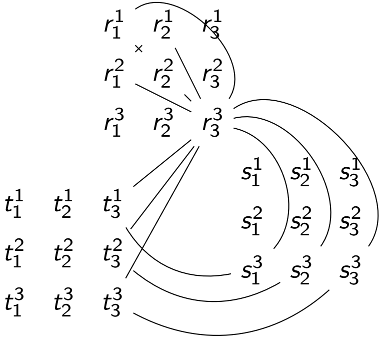

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Thank you!

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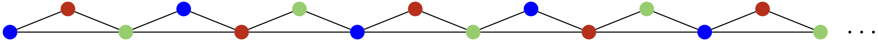

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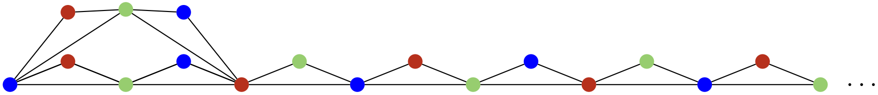

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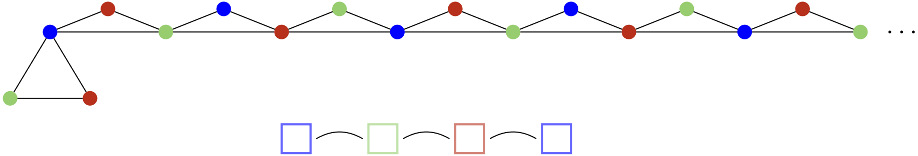

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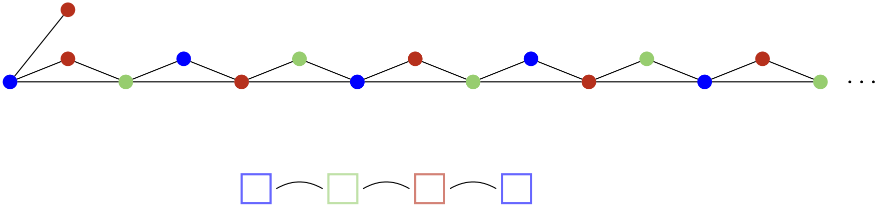

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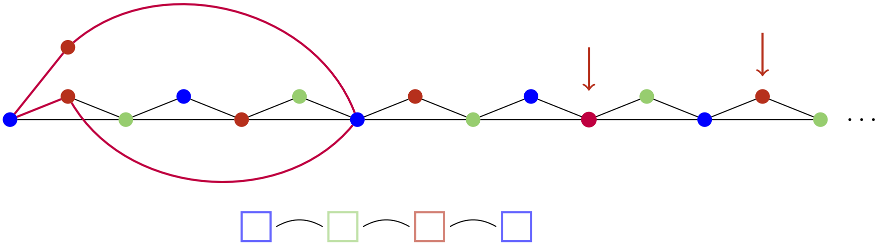

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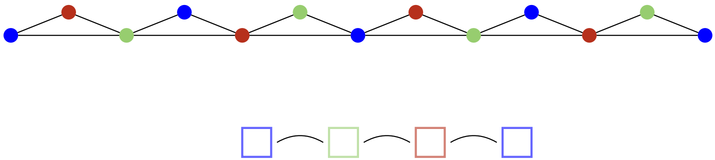

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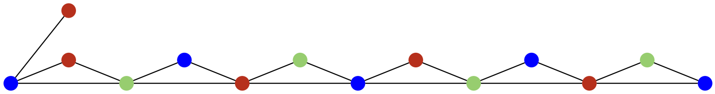

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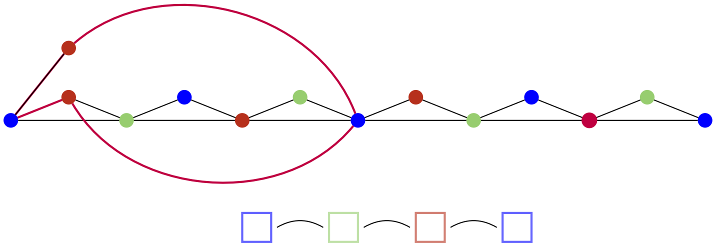

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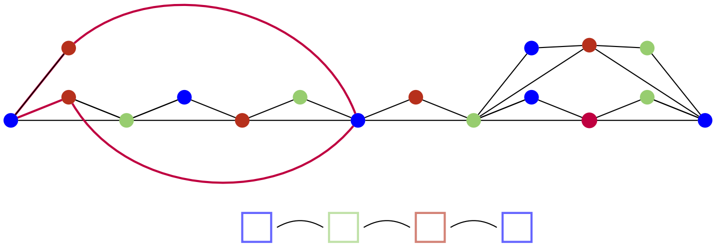

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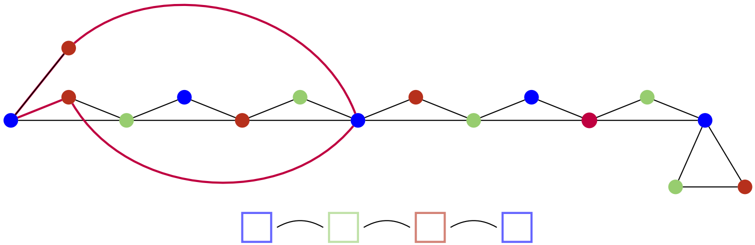

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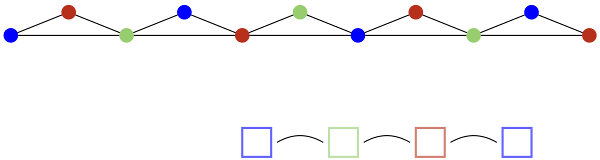

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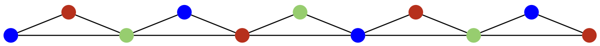

Lemma

Length at most 6 $\Leftrightarrow$ Hitting set of size at most 6.

Hitting set of a co-bridge free Path of triangles graphs

Lemma

Co-bridge free paths of triangles graphs have a hitting set of size at most 5.

Hitting set of a co-bridge free Path of triangles graphs

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- Length $= 6$,

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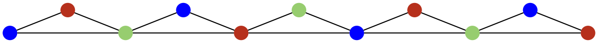
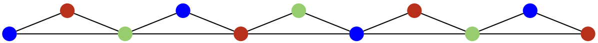


Hitting set of a co-bridge free Path of triangles graphs

Lemma

Co-bridge free paths of triangles graphs have a hitting set of size at most 5.

- Length = 6,
- we can only add "pending" triangles.



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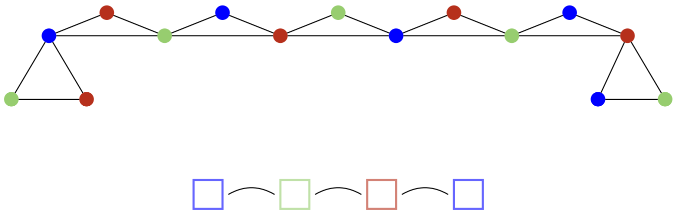

Hitting set of a co-bridge free Path of triangles graphs

Lemma

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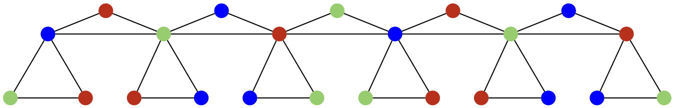
- all sets have "pending" triangles.



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